Uncertainty of the estimation of resonant transmission loss below the critical frequency

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Direct measurement of the flanking sound transmission coefficients in each direction

- There have been three main ways proposed to predict the flanking sound transmission coefficient for a particular path
- The first is to measure the actual flanking sound transmission coefficient for a particular path for the particular construction by covering the other paths and establishing a database of measured values
- Because it is believed that the flanking transmission coefficients should be the same in both directions, the geometric mean of the flanking transmission coefficients can be taken
- This method is being followed by the National Research Council of Canada
- At the FP0702 meeting in Bruges it was decided that FP0702 would <u>NOT</u> follow this approach
- The second method is the measurement or prediction of the airborne excited and the mechanically excited radiation efficiencies and the direct sound transmission coefficients

Measurement or prediction of airborne and mechanically excited radiation efficiencies

- $$\tau_{ij} = \tau_i d_{ij} \frac{\sigma_{mj}}{\sigma_{ai}} \frac{S_j}{S_0}$$ Gerretsen (1979)
- τ_{ij} is the flanking sound power transmission coefficient via elements i and j
- τ_i is the direct sound power transmission coefficient of element i
- d_{ij} is the ratio of the mean square velocities in elements j and i
- σ_{ai} is the radiation efficiency of the element i when excited by airborne sound
- σ_{mj} is the radiation efficiency of the element j when excited mechanically
- S_j is the area of element j
- S_0 is area of the common element between the two rooms
- The CSTB proposal is to use this equation
- This requires the measurement or prediction of the airborne excited and the mechanically excited radiation efficiencies and the direct sound transmission coefficients

Direction averaged sound transmission coefficient

- Because it is believed that the flanking transmission coefficients should be the same in both directions, the geometric mean of the flanking transmission coefficients can be taken

$$\tau_{ij} = \tau_{ji} = \tau_{ijav} = \sqrt{\tau_i \tau_j}\sqrt{d_{ij} d_{ji}}\sqrt{\frac{\sigma_{mi}}{\sigma_{ai}}\frac{\sigma_{mj}}{\sigma_{aj}}}\frac{\sqrt{S_i S_j}}{S_0}$$

- This equation should be used to calculate the flanking transmission
- Above the critical frequency of an element, the airborne and mechanically excited radiation efficiencies of the element are equal
- Thus above the critical frequency, the above equation reduces to

$$\tau_{ij} = \tau_{ji} = \tau_{ijav} = \sqrt{\tau_i \tau_j}\sqrt{d_{ij} d_{ji}}\frac{\sqrt{S_i S_j}}{S_0}$$

- Gerretsen (1979)

The non-resonant and resonant sound transmission coefficients and velocities

- In this presentation, the forced or non-resonant sound transmission coefficient and the forced or non-resonant velocity are defined to be values that would occur if the panel was completely limp and was excited by a diffuse incidence sound field
- This is different from the values calculated by Sewell, Rudder, Leppington *et al.* and Lee and Ih
- These authors include Sewell's factor of

$$\frac{1}{\left[1-\left(\frac{\omega}{\omega_c}\right)^2\right]^2}$$

- which accounts for the approach of the angular frequency ω to the critical angular frequency ω_c
- The resonant sound transmission coefficient and resonant velocity are the difference between their total values and their forced or non-resonant values when excited by a diffuse incidence sound field

Forced and resonant vibration response

- The mean squared velocity v^2 of a diffuse airborne field excited homogeneous isotropic finite panel is the sum of the forced or non-resonant mean square velocity v_f^2 and the resonant mean square velocity v_r^2

$$v^2 = v_f^2 + v_r^2$$

- The radiated sound intensity on one side of the panel is

$$I = v_f^2 \sigma_f + v_r^2 \sigma_r = v^2 \sigma_a$$

- where σ_r and σ_f are the resonant and forced radiation efficiencies
- Crocker and Price (1969) and others have shown that

$$\frac{v_r^2}{v_f^2} = \frac{\pi \omega_c \sigma_r}{4 \omega \eta} = r$$

- ω and ω_c are the angular and angular critical frequencies, η is the total damping loss factor and r denotes the value of the ratio

Radiation efficiency ratio

- The equations on the previous slide can be solved to produce

$$\sigma_a = \frac{r\sigma_r + \sigma_f}{r+1}$$

- and

$$\frac{\sigma_r}{\sigma_a} = \frac{r+1}{r + \sigma_f / \sigma_r}$$

- If the radiation from the near field bending waves at the point or line mechanical source is ignored, then $\sigma_m = \sigma_r$ and

$$\frac{\sigma_m}{\sigma_a} = \frac{r+1}{r + \sigma_f / \sigma_r}$$

- This equation can be used to calculate the flanking transmission if we know the ratio of the resonant to the forced mean square velocity, the ratio of the forced to resonant radiation efficiency and the direct sound transmission coefficient

Resonant sound transmission coefficients

- The third method is the use of the resonant sound transmission coefficients
- If r >> 1, then $v^2 \approx v_r^2$ and their value can be determined from the resonant sound transmission coefficients (Gerretsen 1979)
- If this assumption is made, our earlier equation becomes

$$\tau_{ij} = \tau_{ji} = \tau_{ijav} = \sqrt{\tau_{ri}\tau_{rj}}\sqrt{d_{ij}d_{ji}}\sqrt{\frac{\sigma_{mi}}{\sigma_{ri}}\frac{\sigma_{mj}}{\sigma_{rj}}}\frac{\sqrt{S_i S_j}}{S_0}$$

- τ_{ri} is the resonant sound transmission coefficient of element i
- If the radiation from the near field bending waves at the point or line mechanical source is ignored, then $\sigma_m = \sigma_r$ and our equation becomes

$$\tau_{ij} = \tau_{ji} = \tau_{ijav} = \sqrt{\tau_{ri}\tau_{rj}}\sqrt{d_{ij}d_{ji}}\frac{\sqrt{S_i S_j}}{S_0}$$

- The problem is how to determine the resonant sound transmission coefficients

Subtraction of forced sound transmission coefficient

- Gerretsen (2007) has suggested subtracting the predicted forced sound transmission coefficient from the measured sound transmission coefficient to obtain the resonant sound transmission coefficient
- A major problem with this method is that below the critical frequency, the resonant sound transmission coefficient is very much smaller than the forced sound transmission coefficient
- Because of the measurement and prediction uncertainties, the calculated value will be very uncertain and may possibly produce non-physical negative values (Mahn 2008)
- Another problem is that the prediction of the “forced” sound insulation of typical lightweight cavity wall constructions is more complex
- Stud walls have significant bending wave near field radiation. Is this forced?
- Even for single homogeneous isotropic walls, different theories produce different predictions
- Does only one leaf of a multi-leaf wall need to be considered?

1.6 mm steel

- Mahn (2008) measured the sound transmission coefficient of a 1.6 mm thick steel panel from 100 Hz to 5 kHz and used various theories to predict the forced sound transmission coefficient and subtracted it from the total sound insulation
- Rudder produced no positive values
- Leppington and Lee produced positive values at 100 to 160 Hz and 200 Hz
- Sewell produced positive values at 100, 160 and 315 Hz
- Gerretsen produced positive values at 100, 160, 315, 2500 and 5000 Hz
- The field incidence mass law produced positive values from 1600 to 5000 Hz
- The normal incidence mass law produced positive values at all frequencies
- The critical frequency was in the 8 kHz band
- The surface density was 12.5 kg/m^2

4 mm MDF

- Mahn (2008) also made measurements on 4 mm MDF
- Rudder produced no positive values
- Leppington, Lee and Sewell produced positive values at 100 Hz
- Gerretsen produced positive values at 100 and 5000 Hz
- The field incidence mass law produced positive values from 3150 to 5000 Hz
- The normal incidence mass law produced positive values from 250 Hz to 5000 Hz
- The critical frequency was in the 8 kHz band
- The surface density was 3.2 kg/m^2

Gerretsen's 2007 measurements

- However Gerretsen (2007) has applied the method without obtaining negative values with octave band measurements from 63 Hz to 4000 Hz
- The materials were 70 mm calcium-silicate (130 kg/m^2), 70 mm gypsum blocks (66 kg/m^2), 10 mm glazing (25 kg/m^2) and 12,5 mm gypsum board with studs (15 kg/m^2)
- Note that these materials all have higher surface densities and lower critical frequencies than Mahn's (2008) materials
- As noted earlier, Sewell, Rudder, Leppington *et al.* and Lee and Ih include Sewell's factor for the approach of the frequency to the critical frequency
- Sewell, Leppington *et al.,* Lee and Ih and Gerretsen use a forced radiation efficiency which varies with frequency

Correction factor

- If τ_f and τ_r are the forced and resonant sound transmission coefficients, the correction factor is

$$C = 1 + \frac{\tau_f}{\tau_r} = 1 + \frac{\sigma_f}{r\sigma_r}$$

- and the resonant sound transmission coefficient can be calculated from

$$\tau_r = \frac{\tau}{C}$$

- Since the ratio of the resonant to the forced mean square velocities and ratio of the forced to the resonant radiation efficiency are needed to calculate the correction factor, there seems to be no advantage to using a correction factor compared to the more exact method proposed by CSTB
- Nevertheless a number of correction factor methods have been proposed
- They differ only in the values for the two ratios that they use

Metzen's correction factor

- This has been proposed by Metzen (2004) and Gerretsen (2007)
- For a square panel, it is equivalent to using the value of r given earlier, a forced radiation efficiency of 1 and the following low frequency approximation to the resonant radiation efficiency given in the first two editions of Cremer and Heckl's textbook

$$\sigma_r = \frac{U\lambda_c}{\pi^2 S}\sqrt{\frac{\omega}{\omega_c}} \text{ for } \omega << \omega_c$$

- U is the perimeter, S is the area, ω is the angular frequency, λ_c is the wavelength of sound in air at the angular critical frequency ω_c
- This expression is a very low frequency approximation to the well known result of Maidanik and Leppington below the critical frequency
- If this correction factor is to be used it would be better to use a more exact diffuse field incidence forced radiation efficiency (see Davy (2009) for a discussion of some possible formulae) and Maidanik and Leppington's result for the resonant radiation efficiency

Annex B of EN 12354 part 1

- Gerretsen (2007) has also proposed a correction factor based on Annex B of EN 12354 part 1
- It is based on Josse and Lamure's (1964) calculations of the forced and resonant sound transmission coefficients
- Assuming Cremer and Heckl's low frequency approximation for the resonant radiation efficiency, this correction factor for a square panel is equivalent to

$$r = \frac{v_r^2}{v_f^2} = \frac{\sigma_r}{\eta}\sqrt{\frac{\omega_c}{\omega}}$$

- The fact that this differs from the generally accepted formula of

$$r = \frac{v_r^2}{v_f^2} = \frac{\pi \omega_c \sigma_r}{4\omega\eta}$$

- suggests that the formula for the resonant transmission in EN 12354 part 1 may be wrong

Annex B of EN 12354 part 1

- $\pi/4$ is fairly close to 1, but appearance of the square root is a concern
- The value of r derived from Annex B of EN 12354 part 1 is, apart from the factor ½, the same as the formula for the ratio of the sound power radiated by the resonant vibration to the sound power radiated by the forced near field vibration for a line source on a panel
- The generally accepted value of r is the same as the value of r when a panel is excited by a point force
- These last two statements suggest that Josse and Lamure may have used an essentially 2D model
- While a 2D would normally be adequate, it may be that a full 3D model is needed to predict the resonant sound transmission loss below the critical frequency
- The formula for the resonant sound transmission loss in Annex B is not immediately obviously derivable from Josse and Lamure's equations

CSTB correction factor

- The CSTB approach gives the following correction factor

$$C = \frac{\tau}{\tau_r} = 1 + \frac{\sigma_f}{\sigma_r} \frac{\sigma_a - \sigma_r}{\sigma_f - \sigma_a}$$

$$\text{If } \sigma_r << \sigma_a << \sigma_f, \; C = \frac{\tau}{\tau_r} \approx \frac{\sigma_a}{\sigma_r}$$

- However, if the information to calculate this correction factor is available, the more direct second method proposed by CSTB can also be used
- Since this direct second method does not assume that the total mean square velocity is equal to the resonant mean square velocity, it appears to be the preferred method.

Nightingale and Bosmans 2003 correction factor

- Nightingale and Bosmans effectively use the Crocker and Price (1969) SEA value of r given earlier in this paper. They quote Craik's 1996 book.
- Maidanik and Leppington's value of the resonant radiation efficiency is used
- Leppington's modification of Sewell's forced radiation efficiency is used
- They follow Leppington and include Sewell's factor to account for the approach to critical frequency from below
- The inclusion of this factor in the forced (non-resonant) sound transmission coefficient was a surprise to this presenter
- In SEA, the effects of this factor are accounted for by the modal response
- It is also surprising that they did not use Leppington's formula for the non-resonant sound transmission coefficient below the critical frequency
- This shows the difficulty of defining the resonant and the forced (non-resonant) sound transmission coefficients

Direct theoretical calculation of the resonant sound transmission coefficient

- Another possibility is direct theoretical calculation of the resonant sound transmission coefficient
- Rindel and Lee and Ih effectively calculate the mean square mass law velocity for unit incident intensity and multiply it by r and the resonant radiation resistance
- Annex B of EN 12354 part 1 can be used to calculate the resonant sound transmission coefficient. Unfortunately, as noted above, this formula appears to wrong.
- Leppington's formula can be used, but as noted above there is some doubt as to whether his separation into resonant and non-resonant parts is appropriate for the purpose of EN 12354. In particular Nightingale and Bosmans only used his non-resonant formula and not his resonant formula.

The difficulty of calculation for complex walls

- Most of the theory given in this paper is only correct for monolithic homogeneous and isotropic walls
- Most lightweight walls are more complex than this simple wall type
- Ribs such as studs and joists complicate the theoretical calculations
- Studs and joists will often provide a structural transmission path between separate wall leaves
- It is not always clear whether the whole wall or just one of its leaves should be used
- Because of the high attenuation rates of velocity with distance that have been observed with lightweight walls, it is not clear how applicable the EN 12354 method is to these lightweight walls

Conclusions

- The airborne sound excited and the mechanically excited radiation efficiencies should be measured as well as the direct sound transmission coefficients
- This data should be used with the direct method to calculate the flanking sound transmission coefficients
- The flanking transmission coefficients should be calculated in both directions
- If the values of the flanking sound transmission coefficients in both directions are not too different, the geometric mean of the flanking sound transmission coefficients should be calculated
- Theoretical predictions of the airborne sound excited and the mechanically excited radiation efficiencies and the direct sound transmission coefficients should be compared with the measured values to aid the development of suitable theoretical prediction methods